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Kantowaski-Sachs Inflationary Universe in General Relativity

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Abstract Kantowaski space time in the presence of mass less scalar field with a flat potential is investigated. To obtain an inflationary universe, we have considered a flat region in which V is constant. Some physical properties of the model are also discussed.

Keywords Kantowaski-Sachs model · Inflationary universe · General relativity

1 Introduction

In recent years there has been lot of interest in inflationary models of the universe in general relativity. Inflationary models play an important role in solving number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The standard explanation for the flatness of the universe is that it has undergone at an early stage of the evolution a period of exponential expansion named as inflation.

It is well known that self interacting scalar fields play a vital role in the study of inflationary cosmology. Guth [8], Linde [11] and La and Steinhart [10] are some of the authors who have investigated several aspects of the inflationary universe in general relativity. While Burd and Barrow [5], Wald [18], Barrow [2], Eills and Madsen [7], Hensler [9], Chakraborty [6] studied different aspects of scalar fields in the evolution of the universe. The role of self-interacting scalar fields in inflationary cosmology has been investigated by Bhattacharjee and Baruah [4], Bali and Jain [1], Rahman et al. [12], Reddy and Naidu [14], Reddy et al. [13]. Very recently Reddy [15] has discussed Bianchi type-V inflationary universe in general relativity. In this paper, we have investigated Kantowaski-Sachs cosmological model in the presence of mass less scalar fields with a flat potential in general relativity. To get a determinate solution, we have considered a flat region in which flat potential V is constant. We

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have also assumed a relation between metric potentials for this purpose. Kantowaski-Sachs models are astro-physically important because they are considered as possible candidates for an early era in cosmology.

2 Metric and Field Equations

We consider the Kantowaski-Sachs metric in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{1}$$

where A and B are functions of cosmic time t only.

Kantowaski-Sachs class of metric represents homogeneous but anisotropically expanding (contracting) cosmologies and provide models where the effects of anisotropy can be estimated and compared with the FRW-class of cosmologies (Thorne [17]).

In the case of gravity minimally coupled to a scalar field $V(\phi)$, the Lagrangian is

$$L = \int \left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}dx^4 \tag{2}$$

Which on variation of L, with respect to dynamical fields, leads to Einstein field equations

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{3}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right]g_{ij} \tag{4}$$

$$\phi^{i}_{;i} = -\frac{dV}{d\phi} \tag{5}$$

where comma (,) and semicolon (;) indicate ordinary and covariant differentiation respectively.

Other symbols have their usual meaning and units are taken so that

$$8\pi G = C = 1.$$

Now the Einstein's field equations (3) for the metric (1) are given by

$$2\frac{B_{44}}{B} + \frac{(B_4)^2}{B^2} + \frac{1}{B^2} = -\left(\phi_4^2 + V(\phi)\right) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -\left(\phi_4^2 + V(\phi)\right) \tag{7}$$

$$2\frac{A_4 B_4}{AB} + \frac{(B_4)^2}{B^2} + \frac{1}{B^2} = \frac{\phi_4^2}{2} - V(\phi) \tag{8}$$

and (5), for the scalar field, takes the form

$$\phi_{44} + \left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\phi_4 = -\frac{dV}{d\phi} \tag{9}$$

Here the subscript 4 denotes differentiation with respect to t.

3 Inflationary Model

We are interested, here, in inflationary solutions of the field equations (6)–(9).

Stein-Schabas [16] has shown that Higgs field ϕ with potential $V(\phi)$ has flat region and the field evolves slowly but the universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Thus, we are interested here, in inflationary solutions of the field equations.

The flat region is considered where the potential is constant i.e.

$$V(\phi) = const = V_0 \text{ (say)} \tag{10}$$

Since the field equations are highly non linear to get determinate solution one can use a special law of variation of Hubble parameter proposed by Berman [3], which yields a constant negative deceleration parameter model of the universe.

Here, we consider only constant negative deceleration parameter defined by

$$q = -\left(\frac{R_{44}R}{R_4^2}\right) = const, \tag{11}$$

where $R = (AB^2)^{\frac{1}{3}}$ is the overall scale factor. Here the constant is taken as negative, since it is an accelerating model of the universe.

Equation (11) gives the solution

$$R = (AB^2)^{\frac{1}{3}} = (at+b)^{\frac{1}{1+q}}, \tag{12}$$

where a and b are constants of integration.

This equation implies that the condition of expansion is $1+q>0$.

Now with the help of (10), the field equations (6)–(9) yield the solution given by

$$\begin{aligned} A &= \exp\{-k_2(at+b)^2\} \\ B &= (at+b)^{\frac{3k_1}{2}} \exp\left\{-\frac{k_2}{2}(at+b)^2\right\} \\ \phi &= (at+b)^{k_3} + \phi_0 \end{aligned} \tag{13}$$

where, we have

$$k_1 = \frac{1}{1+V}, \qquad k_2 = \frac{V_0}{2e^2}\left(\frac{1+q}{4+q}\right), \qquad k_3 = \frac{q-2}{q+1} \tag{14}$$

Now the metric (1), with the help of (13), after a suitable transformation of coordinates and constants becomes

$$ds^2 = dT^2 - \exp\{-2k_2T^2\}dr^2 - T^{3k_1}\exp\{k_2T^2\}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{15}$$

where k_1, k_2 and k_3 are given by (14).

4 Physical Properties

The cosmological model given by (15) represents Kantowaski-Sachs inflationary universe in general relativity. The model has no initial singularity. The physical and kinematical properties for the model (15) have the following expressions.

$$\begin{aligned}
\text{Spatial volume } &= T^{\frac{3}{1+q}} \\
\text{Expansion scalar } &= \theta = \frac{3}{T(1+q)} \\
\text{Shear scalar } &= \sigma^2 = \left\{\left[\frac{V_0}{2}\left(\frac{1+q}{4+q}\right)\right]T + \frac{1}{T(1+q)}\right\}^2 \\
\text{Hubble parameter } &= H = \frac{1}{(1+q)T}
\end{aligned}$$

The spatial volume increases with time T, since $1+q>0$ and it becomes infinite for large values of T. Thus inflation is possible for large T.

It can be observed that for large T, the parameters θ, σ, H vanish and they diverge when $T \to 0$.

The Higg's field ϕ becomes constant for $T=0$ or $q=2$ and for large values of T it diverges.

Also

$$\frac{\sigma^2}{\theta} = \frac{\{[\frac{V_0}{2}(\frac{1+q}{4+q})]T + \frac{1}{T(1+q)}\}^2}{3}(1+q)T,$$

which becomes infinitely large for large T and hence the model does not approach isotropy.

5 Conclusion

Kantowaski-Sachs cosmological models play an important role in understanding the early stages of evolution of the universe. Here we have found an inflationary Kantowaski-Sachs universe in general relativity. This study will throw some light on the structure formation of the universe, which has astrophysical significance. The model obtained is non singular, expanding and does not approach anisotropy at late times.

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